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Steady-state behavior in atomic three-level Λ and ladder systems with incoherent population pumping

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We study the steady state in three-level lambda (Λ) and ladder systems. It is well known that in a Λ system this steady state is the coherent population trapping state, independent of the presence of spontaneous emission. In contrast, the steady state in a ladder system is in general not stable against radiative decay and exhibits a minimum in the population of the ground state. We show that incoherent population pumping destroys the stability of the coherent population trapping state in the Λ system and suppresses a previously discovered sharp dip in the steady-state response. In the ladder system the observed minimum disappears in the presence of an incoherent pump on the upper transition. [S1050-2947(97)04003-1]

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The first observation in an optical pumping experiment of what is now known as the phenomenon of coherent population trapping (CPT) was made in 1976 by Alzetta *et al.* [1]. They found that the fluorescence intensity of sodium vapor, illuminated by a laser and analyzed as a function of an applied magnetic field, decreased when the difference in frequency of two laser modes matched some hyperfine transitions of the ground state of sodium. A theoretical explanation for this observation can be given in terms of a three-level atomic system in which coherent radiation fields couple two ground states, the initial and final state, to a common excited state. Under the two-photon Raman resonance condition, when the frequency difference of the radiation fields equals the separation between the initial and final levels, population becomes trapped in a coherent superposition of these two levels. Since its discovery, many studies of CPT have appeared using three-level Λ , ladder, and V systems [2–6]. In addition, the concept of CPT has found interesting applications in, for example, the fields of laser cooling [7], adiabatic population transfer [8], and lasing without inversion [9–12].

Recently, Jyotsna and Agarwal [13] studied the dynamics of the populations in a three-level Λ system with spontaneous emission [cf. Fig. 1(a)]. They demonstrated that the steady state reached by the system is always the CPT state, irrespective of the strengths of the fields and the rates of spontaneous emission. For weaker fields it only takes longer times to reach the CPT state. Motivated by this result, we investigate here the influence of incoherent population pumping on one of the two transitions in the Λ system. It will be shown that in the presence of this additional pumping mechanism, the steady state is no longer the CPT state and that a sufficiently strong rate of incoherent pumping suppresses the dip that was found in the steady-state response of a Λ system as a function of detuning from the two-photon resonance condition [13]. Besides, we show that incoherent population pumping has an interesting effect on the steady-state population in another three-level configuration, the ladder system [Fig. 1(b)]. It is well known that in the absence of radiative decay the ladder system also gives rise to coherent population trapping [3]. Spontaneous emission from the up-

per level destroys this trapping behavior and leads to a minimum in the population of the lower level. We demonstrate that the minimum disappears in the presence of an incoherent pump.

Consider the three-level Λ system schematically depicted in Fig. 1(a).

Two states $|2\rangle$ and $|3\rangle$ are coupled to state $|1\rangle$ via classical monochromatic light fields of frequencies ω_2 and ω_3 , respectively. Level $|1\rangle$ decays to the lower-lying level $|2\rangle$ ($|3\rangle$) by spontaneous emission with rate $2\gamma_2$ ($2\gamma_3$). Let Ω_2 and Ω_3 , defined as $\Omega_j = \mu_{1j}\mathcal{E}_j/\hbar$ ($j=2,3$), denote the Rabi frequencies associated with the coupling by the fields, with μ_{1j} the dipole matrix interaction term between level 1 and level j , and \mathcal{E}_j the amplitude of laser field j . The Hamiltonian for this system in the appropriate rotating frame can be written as

$$\mathcal{H} = \hbar\Delta_3|1\rangle\langle 1| + \hbar(\Delta_3 - \Delta_2)|2\rangle\langle 2| - \left(\frac{1}{2}\hbar\Omega_2|1\rangle\langle 2| + \frac{1}{2}\hbar\Omega_3|1\rangle\langle 3| + \text{H.c.} \right), \quad (1)$$

with $\Delta_2 = E_1 - E_2 - \hbar\omega_2$ and $\Delta_3 = E_1 - \hbar\omega_3$. We have chosen the energy of level $|3\rangle$, E_3 , as the zero of energy. The density matrix equations are obtained from the Liouville equation including damping effects [14] and have partly been described elsewhere [9,11,13]. In the presence of an incoherent pump [15] of rate λ on the $|1\rangle$ - $|2\rangle$ transition they are given by

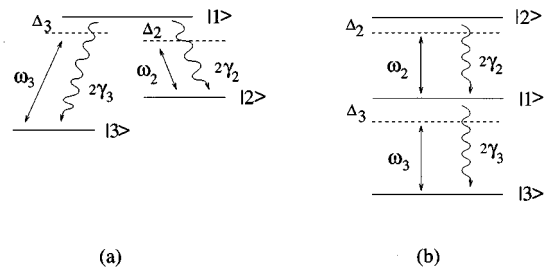


FIG. 1. Schematic representation of a three-level Λ (a) and three-level ladder (b) atomic system.

$$\dot{\rho}_{11} = -2(\gamma_2 + \gamma_3)\rho_{11} - \left(\frac{i}{2}\Omega_2^*\rho_{12} + \frac{i}{2}\Omega_3^*\rho_{13} + \text{H.c.} \right) - 2\lambda(\rho_{11} - \rho_{22}), \quad (2a)$$

$$\dot{\rho}_{12} = -(\gamma_2 + \gamma_3 - i\Delta_2)\rho_{12} + \frac{i}{2}\Omega_2(\rho_{22} - \rho_{11}) + \frac{i}{2}\Omega_3\rho_{32} - 2\lambda\rho_{12}, \quad (2b)$$

$$\dot{\rho}_{13} = -(\gamma_2 + \gamma_3 - i\Delta_3)\rho_{13} + \frac{i}{2}\Omega_2\rho_{23} + \frac{i}{2}\Omega_3(1 - 2\rho_{11} - \rho_{22}) - \lambda\rho_{13}, \quad (2c)$$

$$\dot{\rho}_{22} = 2\gamma_2\rho_{11} + \left(\frac{i}{2}\Omega_2^*\rho_{12} + \text{H.c.} \right) + 2\lambda(\rho_{11} - \rho_{22}), \quad (2d)$$

$$\dot{\rho}_{23} = i(\Delta_3 - \Delta_2)\rho_{23} + \frac{i}{2}\Omega_2^*\rho_{13} - \frac{i}{2}\Omega_3\rho_{21} - \lambda\rho_{23}. \quad (2e)$$

For $\Delta_2 = \Delta_3 = 0$ and $\Omega_2 = \Omega_3 \equiv \Omega$ real, this leads to the steady-state solution

$$\rho_{11} = \frac{\Omega^2\lambda}{N}, \quad (3a)$$

$$\rho_{22} = \frac{\Omega^2 T}{2NM}, \quad (3b)$$

$$\rho_{33} = 1 - \rho_{11} - \rho_{22}, \quad (3c)$$

$$\text{Im}(\rho_{12}) = \frac{2}{\Omega}[(\gamma_2 + \lambda)\rho_{11} - \lambda\rho_{22}], \quad (3d)$$

$$\text{Im}(\rho_{13}) = \frac{2\gamma_3}{\Omega}\rho_{11}, \quad (3e)$$

$$\text{Re}(\rho_{23}) = \rho_{22} - \frac{(\gamma_2 + \gamma_3 + \lambda)}{\lambda}\rho_{11}, \quad (3f)$$

$$\text{Re}(\rho_{12}) = \text{Re}(\rho_{13}) = \text{Im}(\rho_{23}) = 0, \quad (3g)$$

where

$$M = 2\lambda(\gamma_2 + \gamma_3 + 2\lambda) + \Omega^2,$$

$$N = (4\gamma_3\lambda + \Omega^2)(\gamma_2 + \gamma_3 + \lambda) + 2\lambda\Omega^2,$$

$$T = (\gamma_2 + \gamma_3 + 2\lambda)(4\lambda(\gamma_2 + \lambda) + \Omega^2).$$

In the absence of the incoherent pump, so $\lambda = 0$, the steady state (3g) equals the CPT state, which has been studied by Jyotsna and Agarwal [13]. In the Λ system with symmetric fields the population is then equally distributed between the two ground levels. Switching on the incoherent pump and taking equal rates of spontaneous emission $\gamma_2 = \gamma_3 \equiv \gamma$, we have plotted in Fig. 2 the steady-state populations ρ_{22}

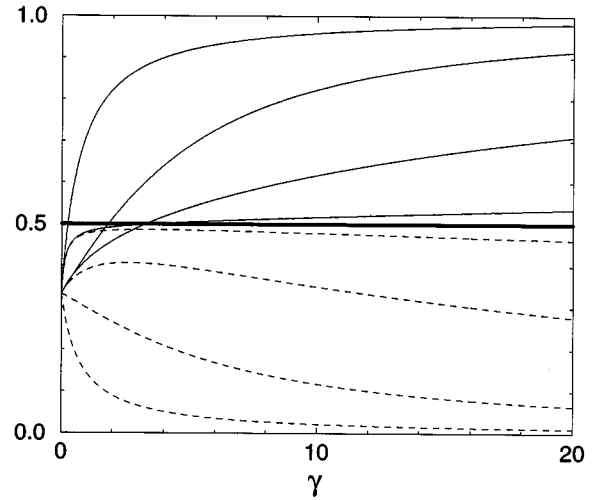


FIG. 2. ρ_{22} (dashed curves) and ρ_{33} (solid curves) in the Λ configuration as a function of $\gamma_2 = \gamma_3 \equiv \gamma$. From top to bottom (bottom to top) the solid (dashed) curves correspond to $\lambda = 100, 10, 1$, and 0.1 , respectively. For $\lambda = 0$, both populations are equal to $1/2$ and independent of γ , which is indicated by the thick line in the middle of the figure. Parameters used are $\Omega_2 = \Omega_3 = 10$ and $\Delta_2 = \Delta_3 = 0$.

(dashed curves) and ρ_{33} (solid curves) as a function of the rate of spontaneous emission for various pumping rates.

As soon as $\lambda \neq 0$, the distribution of population over the two ground states becomes asymmetric ($\rho_{33} > \rho_{22}$) and the higher λ , the larger the difference between the two. In fact, as $\gamma \rightarrow \infty$, all curves for ρ_{33} go to 1, and those for ρ_{22} go to 0, except if $\lambda = 0$. For sufficiently strong decay rates, the incoherent pump thus depletes level $|2\rangle$. This can be seen more clearly in Fig. 3, which shows ρ_{22} on a larger scale for γ .

From Eq. (3) we obtain in the limit $\gamma\lambda \gg \Omega^2$ that

$$\rho_{11} \approx \frac{\Omega^2}{4\gamma(2\gamma + \lambda)} \approx 0,$$

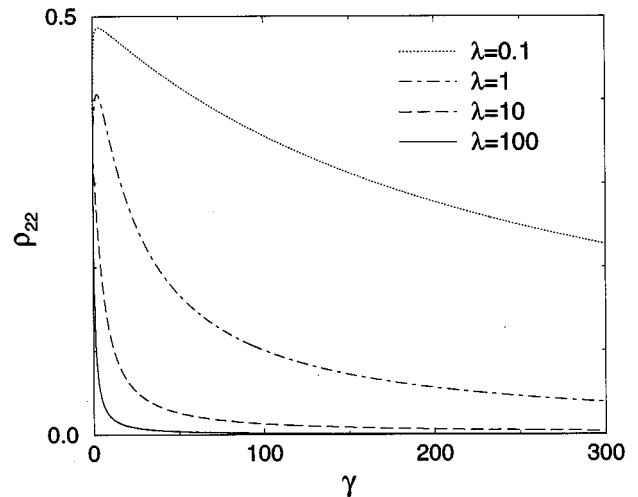


FIG. 3. ρ_{22} in the Λ system as a function of $\gamma_2 = \gamma_3 \equiv \gamma$. Parameters used are $\Omega_2 = \Omega_3 = 10$ and $\Delta_2 = \Delta_3 = 0$.

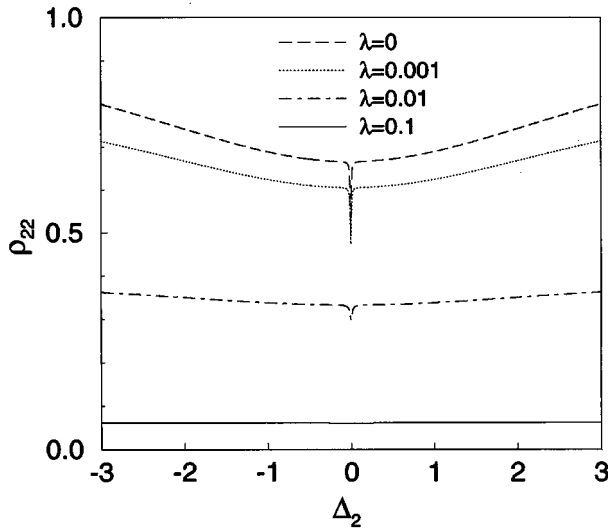


FIG. 4. The population of level $|2\rangle$ in the Λ system as a function of detuning Δ_2 for various incoherent pumping rates λ . The parameters used are $\Omega_2 = \Omega_3 = 0.2$, $\gamma_2 = 2.0$, $\gamma_3 = 1.0$, and $\Delta_3 = 0$.

$$\rho_{22} \approx \frac{\Omega^2(\gamma + \lambda)}{4\gamma\lambda(2\gamma + \lambda)} \approx 0,$$

$$\rho_{33} \approx \frac{4\gamma\lambda(2\gamma + \lambda) - \Omega^2(\gamma + 2\lambda)}{4\gamma\lambda(2\gamma + \lambda)} \approx 1.$$

Figure 2 also shows that $\rho_{11} = \rho_{22} = \rho_{33} = 1/3$ at $\gamma = 0$, as long as $\lambda \neq 0$. In the absence of decay, the incoherent pump distributes the population equally among the levels of the Λ system, independent of λ and Ω .

Armed with the knowledge that incoherent population pumping leads to a steady state that is different from the CPT state in the Λ system, one could ask how it would affect the dependence of ρ_{22} on the detuning Δ_2 . The question arises because Jyotsna and Agarwal [13] found that for weak fields and unequal decay rates γ_2 and γ_3 , ρ_{22} exhibits a sharp dip as a function of Δ_2 around the Raman resonance condition $\Delta_2 = \Delta_3$. They explain that this happens because for unequal decay rates (say $\gamma_2 > \gamma_3$), more population is present in state $|2\rangle$ than in state $|3\rangle$ if $\Delta_2 \neq \Delta_3$. But at the resonance condition, state $|2\rangle$ always contains half of the population. Hence a dip occurs around this value, which becomes sharper with increasing γ_2 . For $\lambda \neq 0$, however, the steady state at $\Delta_2 = \Delta_3$ is no longer the CPT state and level $|2\rangle$ contains less than half of the total population. We have calculated the steady-state response of the Λ system with incoherent pumping for $\Delta_2 \neq \Delta_3$. The resulting expressions are lengthy and therefore not given here. However, the dependence of ρ_{22} on Δ_2 for weak fields $\Omega \ll \gamma_2, \gamma_3$ is shown in Fig. 4.

We see an overall decrease in ρ_{22} as λ increases. The dip in ρ_{22} completely disappears for $\lambda \approx \Omega$ and ρ_{22} then becomes independent of the detuning. This is expected if incoherent pumping is the dominant pumping mechanism, since the amount of detuning from the transition is irrelevant for a pump with a linewidth much larger than the transition width. The same behavior occurs for stronger fields, for which the dip is smoother [13].

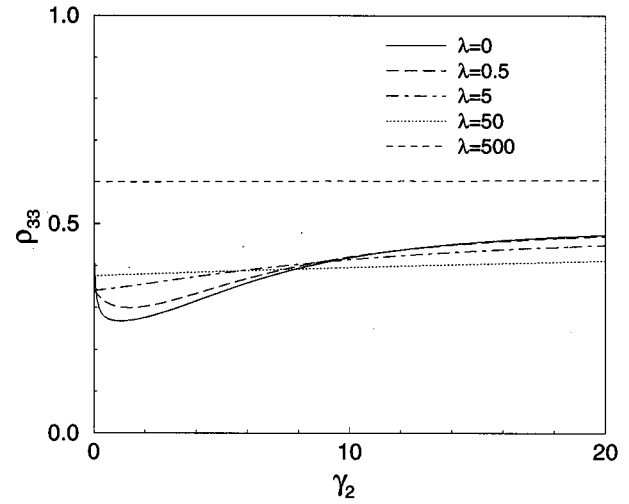


FIG. 5. ρ_{33} vs γ_2 in the ladder system for increasing incoherent pumping rates on the $|1\rangle$ - $|2\rangle$ transition. The parameters used are $\Omega_2 = \Omega_3 = 10$, $\Delta_2 = \Delta_3 = 0$, and $\gamma_3 = 0.1$.

Let us now consider the ladder system of Fig. 1(b) with an incoherent pump on the $|1\rangle$ - $|2\rangle$ transition. The Hamiltonian and the evolution equations of the reduced density matrix for this system are similar to those of the Λ configuration and not given here (see, e.g., [10,11]). From them the steady-state solution is easily obtained. Figure 5 shows the steady state population of level $|3\rangle$ as a function of γ_2 for symmetric fields and under the Raman resonance condition.

We see that ρ_{33} exhibits a minimum in the absence of incoherent population pumping. This minimum arises because as soon as $\gamma_2 \neq 0$, the steady state in the ladder system deviates from the CPT state and level $|1\rangle$ becomes populated. For small values of γ_3 [16], level $|3\rangle$ contributes to the population in level $|1\rangle$ and so ρ_{33} initially decreases with γ_2 . However, as γ_2 increases further, ρ_{33} approaches again the value $1/2$ and hence the minimum is formed.

The size of the minimum becomes smaller with increasing λ and disappears for $\gamma_3 \ll \lambda \leq \Omega$. If the rate of incoherent pumping dominates the spontaneous emission from level $|2\rangle$, one can show that

$$\rho_{11} \approx \frac{\Omega^2}{4\lambda\gamma_3 + 3\Omega^2},$$

$$\rho_{22} \approx \frac{\Omega^2}{4\lambda\gamma_3 + 3\Omega^2},$$

$$\rho_{33} \approx \frac{4\lambda\gamma_3 + \Omega^2}{4\lambda\gamma_3 + 3\Omega^2}.$$

The steady-state populations have become independent of γ_2 in this limit and the upper two levels contain equal amounts of population, as expected when the coupling between these levels is large compared to the decay.

Summarizing, we analyzed the behavior of the steady state in atomic three-level Λ and ladder systems in the presence of incoherent population pumping. In the Λ system the steady state is no longer equal to the CPT state under influence of this pump. A similar decay of the trapping state has recently been shown to occur due to fluctuations between two driving fields in a double- Λ configuration [17]. We have demonstrated that a sufficiently strong rate of incoherent

pumping suppresses the sharp dip that was found in the steady-state response of the Λ system as a function of detuning Δ_2 [13]. In the ladder configuration, the minimum that occurs in the steady-state population of the lowest level $|3\rangle$ as a function of the rate of spontaneous emission from the upper level $|2\rangle$ disappears if the $|1\rangle$ - $|2\rangle$ transition is incoherently pumped.

Other works on three-level atomic level schemes have demonstrated that phase diffusion in the pumping fields has substantial effects on coherence phenomena such as the gain in a lasing without inversion (LWI) ladder system [18] and the refractive index enhancement in a V configuration [19]. These phase fluctuations in the driving fields are also known

to lead to a decay of the CPT state in Λ systems [4,17] and thus their effect seems to be qualitatively similar to that of incoherent population pumping. Another point of consideration is that we have only studied dilute media of Λ and ladder systems here. In order to treat dense media, local-field corrections have to be taken into account. It has been shown for the coherently pumped Λ system that CPT persists in dense media for all field strengths [20] and it would be interesting to see what the result is of including an incoherent pump.

I have benefited from useful discussions with G. Nienhuis.

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